Optics Communications 283 (2010) 3255–3258

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Effects of gravity on the shape of liquid droplets

Hongwen Ren [a,*], Su Xu [b], Shin-Tson Wu [b]

[a] Department of Polymer Nano-Science and Engineering, Chonbuk National University, Chonju, Chonbuk, 561-756, Republic of Korea
[b] College of Optics and Photonics, University of Central Florida, Orlando, Florida 32816, USA

ARTICLE INFO

Article history:
Received 9 February 2010
Accepted 19 April 2010

Keywords:
Liquid droplet
Droplet surface
Surface tension
Focal length
Gravity effect

ABSTRACT

We report the gravity effects on the shape and focal length changes of liquid droplets. As the droplet size decreases, the gravity effect is gradually weakened. Moreover, if the outside space of the droplet is filled with another immiscible liquid rather than air, the filled liquid helps to offset the gravity effect even though their densities do not match well. Good agreement between experiment and simulation is obtained. The negligible gravity effect enables us to improve the optical performances of the liquid lens by choosing suitable liquids without worrying their density mismatch.

1. Introduction

The controllable surface profile of a liquid by an external voltage is attractive not only for fundamental research but also for practical applications. Several theoretical frameworks have been published [1–3]. Among them, electrowetting [4–8] and dielectrophoretic effects [9–12] are the most common principles governing the liquid's shape change induced by an external voltage. In a normal condition, the shape of a liquid droplet exhibits axial symmetry so that it behaves like a lens. Nowadays, liquid has become a favored material for fabricating adaptive lenses due to its compact structure, broadband, high transmission, and simple fabrication. Various adaptive liquid lens and microlens array have been developed for imaging processing [5,7,10], beam steering [8,13], optical communications [14], and other photonic applications.

In an electrowetting or dielectrophoresis lens, two immiscible liquids are employed. One liquid forms a droplet on substrate surface and another liquid is used to fill the surrounding space. By applying a voltage to the cell, the shape of the droplet is varied. Therefore, the droplet functions as an adaptive lens. When a small droplet is placed on a horizontal surface, the droplet shape will not be deformed by the gravity force because the axis of the shape is along the direction of gravity force. However, when the lens is placed in vertical direction the gravity effect could deform the shape of the lens. To overcome this undesirable effect, density matching of the two liquids is commonly employed. Many liquids with nice physical properties cannot be used because of the mismatched density.

According to Vafaei and Podowski's theoretical model [15], when a liquid droplet placed on a horizontal surface is small enough the gravity effect on the droplet shape is negligible. Such a result is interesting because the surface force of the droplet can overcome the gravity effect without changing the droplet shape. However, less is known about the gravity effect on the droplet shape if the droplet is placed in vertical direction. A better understanding on how the gravity affects the droplet shape would allow us to improve the liquid lens performances by choosing suitable liquid materials.

In this paper, we experimentally studied the effects of gravity on the shape of various liquid droplets. These droplets were placed in three extreme conditions: on a horizontal surface, below a horizontal surface, and in vertical direction. Our results show that when a droplet size is small enough, the gravity effect on deforming its shape is negligible. Moreover, if the surrounding of the droplet is filled with another liquid, then the liquid helps to reduce the gravity effect. Such results are in good agreement with theoretical simulations. Based on our results, it is possible to prepare a high performances liquid lens by choosing suitable liquid materials without the concern of density mismatching.

2. Fabrication method

To prepare various liquid droplets, we chose optical oil SL-5267 (refractive index $n = 1.67$, surface tension $t \sim 50$ mN/m) as the droplet material. The oil is highly clear with a relatively high density $\rho \sim 1.26$ g/cm^3. To obtain circular-shaped droplets, we mixed the oil with a solvent such as acetone. The solvent served for two purposes: it significantly decreased the surface tension of the oil and it diluted the oil concentration in the mixture. When a small amount of the mixture was dripped on a glass substrate surface, we then heated the mixture

* Corresponding author. Tel.: +83 407 823 4763.
E-mail addresses: hongwen@jbnu.ac.kr (H. Ren), swu@mail.ucf.edu (S.-T. Wu).

0030-4018/$ – see front matter
doi:10.1016/j.optcom.2010.04.045

to 70 °C in order to accelerate the solvent evaporation. After evaporation, various oil droplets were formed due to the coalescing phenomenon. The surface of the substrate was coated with a very thin polyimide layer (PI2556, from HD Microsystems) which was used to anchor the formed droplets. The surface tension of the polyimide is ~40 mN/m.

3. Measurement method

Usually when the shape of a droplet is altered, its associated focal length will change accordingly. To evaluate the impact of gravity on the shape of a droplet, we used an optical microscope to measure the focal length of the droplet under different conditions. Fig. 1 shows the experimental method. The focal length of the droplet can be determined by observing the image of an object through the liquid droplet.

From bottom up the white incident light passes through an object, a droplet sample, and then reaches an object lens. An eyepiece (not shown), located above the object lens, was used to observe the images of the droplet. The distance between the object and the sample is ~12 cm, which is considered to be at infinity in comparison to the liquid droplet. First, we placed a droplet on the horizontal surface, as shown in Fig. 1(a), and then adjusted the distance between the object lens and the sample so that we could get two sharp images. One image is the droplet's top surface and the other is the image of the object. We could also place the droplet upside down, as Fig. 1(b) shows. In this case, we can still get two sharp images. Let us take a droplet with aperture D = 590 μm placed on a substrate surface as an example. Two clear images were taken through this droplet using a digital camera mounted on the microscope, as shown in Fig. 2. The two images are the top droplet surface and a resolution target bar.

4. Results and discussion

The distance that the droplet traveled for getting these two images was measured to be ~665 μm. This distance is equal to the sample's focal length. From Fig. 1(a), the gravity force is working against the inside pressure force of the droplet, therefore the droplet on the horizontal surface is in the maximum relaxed state. To measure the gravity effect, a convenient method is to turn the droplet upside down, as shown in Fig. 1(b). Under such a circumstance, the gravity force and the inside pressure force of the droplet are in the same direction, therefore the droplet beneath the horizontal surface is in the maximum stretched state. Considering the thickness (t) of the liquid droplet, we need to subtract the distance between the two

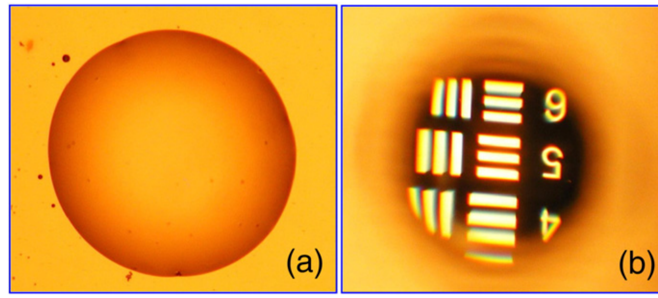

Fig. 2. Measured image of (a) droplet top surface, and (b) resolution bar.

principle pp′ planes from the measured front focal length (f_{front}). Therefore, the focal length of the droplet facing downward (Fig. 1(b)) is expressed as:

$$f_{down} = f_{front} - \frac{n-1}{n} t, \tag{1}$$

where n is the refractive index of the liquid droplet. Using the same measurement method and Eq. (1), the focal length of the droplet facing down was found to be f_{down} ~ 575 μm. The focal length of the droplet facing down from the substrate surface (f_{down}) is shorter than that of facing up (f_{up}). Due to the gravity force, the droplet on the horizontal surface exhibits a minimum curvature and the droplet facing down exhibits a maximum curvature. Thus, the two focal lengths have the following general relationship if gravity takes effect:

$$f_{down} < f_{up} \tag{2}$$

To easily find the effect of gravity on the droplet shape, we use a relative variable range of the focal length to express the focal length change:

$$\frac{\Delta f}{f_{up}} = \frac{f_{up} - f_{down}}{f_{up}}. \tag{3}$$

Fig. 3 shows the measured variable range of the focal length versus various droplets. As the droplet size decreases, $\Delta f / f_{up}$ decreases almost linearly. When the aperture of droplet is smaller than 200 μm, the measured f_{up} is very close to the measured f_{down}. This implies the effect of gravity on the droplet shape is negligible.

For practical applications, a dielectric liquid lens usually employs two liquids in the lens cell. To prepare a dielectric lens and study its gravity effect, we intentionally chose water as the second liquid

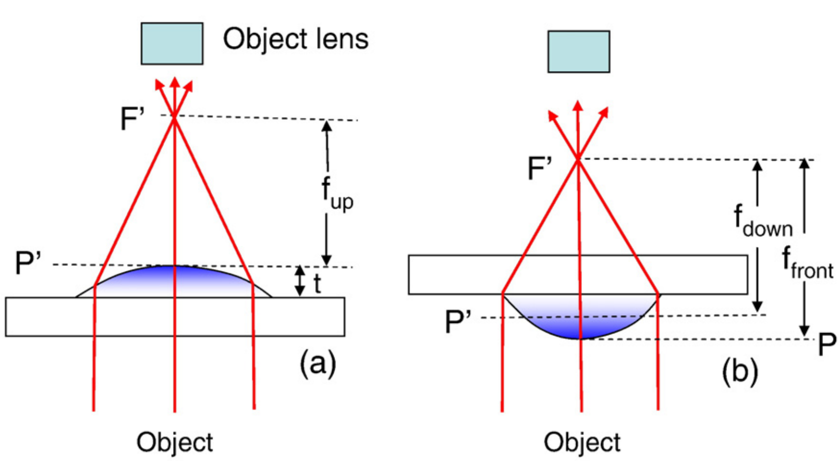

Fig. 1. Method for measuring the gravity effect on the shape of droplet by placing the droplet: (a) on horizontal surface, and (b) with upside down.

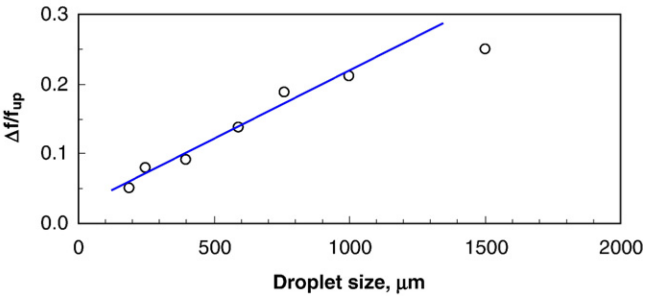

Fig. 3. The size of each droplet versus its relative focal length change $\Delta f/f_{up}$.

because water is immiscible with the oil and its density (1 g/cm^3) has a large mismatch to that of the oil. Since water is much denser than air, the gravity effect on the shape of oil droplet will be decreased with the sustention of water. In this experiment, we filled the surrounding space of the oil with pure water and the two liquids are sealed using another flat glass plate. We then measured the focal length of different droplets which were placed either on or beneath a horizontal substrate surface.

Fig. 4 shows the measured $\Delta f/f_{up}$ versus various droplet sizes. As the droplet size (D) decreases, $\Delta f/f_{up}$ decreases accordingly. When D ~ 665 μm, $\Delta f/f_{up}$ ~ 0.04. As D is further reduced to ~470 μm, the measured f_{up} is almost equal to f_{down}. These results have the same tendency as those shown in Fig. 3, except that the size of the droplet is larger (470 μm vs. 190 μm).

Because the gravity force is in the vertical direction, the shape of each droplet on the horizontal surface keeps the geometrical symmetry without deformation. Those liquid droplets exhibit either spherical or parabolic shape. However, less is known when a droplet is placed in vertical direction. Under such a circumstance, the gravity effect on deforming the droplet shape is maximal. According to Sugiura and Morita's approach [16], the shape of a vertically positioned elastic membrane liquid lens can be expressed as:

$$z = -\frac{\rho g a}{T}\left[\left(y^2 - \frac{y^3}{6a} - \frac{4}{3}ay\right) + k\left(y^2 - 2ay\right)\right], \quad (4)$$

where T is the surface tension of the elastic membrane, ρ the density of the liquid, g the acceleration of gravity, $2a$ the aperture of the lens, and k is the introduced parameter that can be written as:

$$k = w\,/\,2\rho g a, \quad (5)$$

where w is the pumping pressure of the liquid. From Eq. (4), k is a key factor affecting the surface deformation of the liquid lens. In comparison to a conventional membrane liquid lens, the shape of our droplet is sustained by the liquid itself through its surface tension (force) rather than the elastic membrane. Because the liquid surface of the droplet functions as the elastic membrane, we can use Eq. (4) to analyze the shape of our small droplets.

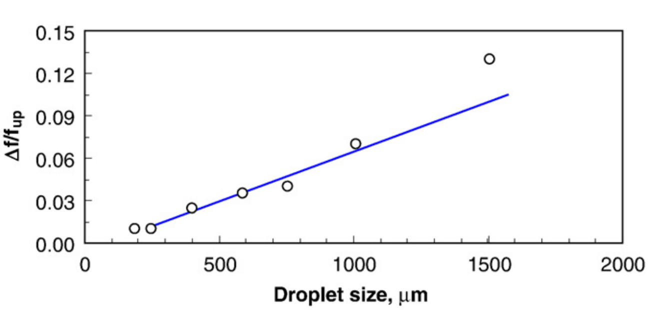

Fig. 4. The size of each droplet surrounded with water versus its relative focal length change $\Delta f/f_{up}$.

The shape of the droplet with aperture $D = 2a = 190$ μm shown in Fig. 3 is gravity-insensitive in horizontal direction, so we chose this droplet for the shape simulation in vertical direction. Fig. 5(a) shows the shape of the droplet with different k values. The apex position of each shape along z-axis was marked using a small open circle.

As k increases, the apex position shifts to the center of the lens. To describe the deformation, the ratio of the shifted distance of the apex point from the lens center and the radius of the lens aperture, called shift ratio, is introduced. The relationship between shift ratio and k value are plotted in Fig. 5(b). As $k > 30$, the shift ratio is saturated (~0.1%). Such a result implies that the gravity effect on the droplet shape is negligible.

We can also estimate the k value of our droplet with aperture $2a = 190$ μm. For this droplet, it bears an inside pressure (w) and an outside pressure (w_o). Their relationship is given by [17]:

$$w - w_o = \frac{2T}{R}, \quad (6)$$

where R is the radius of the droplet curvature. From Fig. 3, the focal length of the droplet is f ~ 250 μm, so $R = f(n_{oil} - 1)$ ~ 167 μm. The surface tension of the droplet is T ~ 50 mN/m, the density of oil ~1.25 g/cm^3, and the outside pressure w_o ~ 1 kg/cm^2. According to Eqs. (5) and (6), we found k ~ 40. From Fig. 5(b), this droplet shape should not be deformed by the gravity force.

To confirm this prediction, we studied the gravity effect on the focusing property of the droplet. In our setup, a collimated He-Ne laser

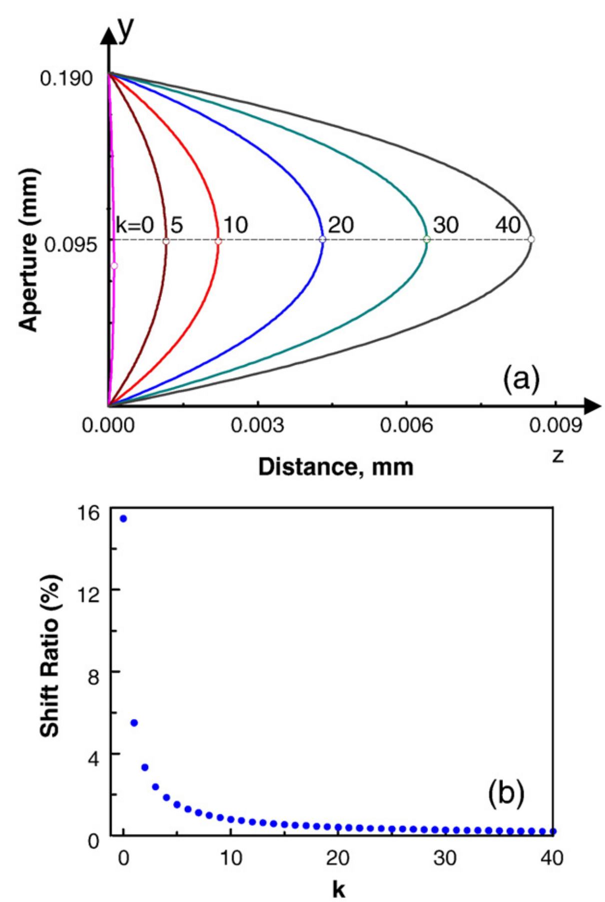

Fig. 5. (a) Simulated surface profiles of the droplet with aperture $D = 190$ μm versus different k values. The small open circles represent the maximum apex points, and (b) the shifted ratio versus k value.

was used to illuminate the droplet. The transmitted light was collected by an imaging lens and detected by a CCD camera (ST-2000XM). The distance between the droplet and the imaging lens was carefully adjusted so that a focused spot pattern was recorded by the CCD camera. All parts were fixed tightly on a small optical table. By rotating the table surface, the droplet could be placed in any direction. If the droplet shape is deformed by the gravity force, then the induced light intensity change will be detected by the very sensitive CCD camera. Fig. 6(a) and (b) show the measured spot intensity profiles (the middle spike) when the droplet was placed in vertical direction and on horizontal glass surface, respectively. The first and third spikes in the figures represent two smaller droplets. The two profiles are almost identical. It means the shape of the droplet does not change although it experiences gravity force in two extreme positions.

Eq. (4) is used to express the shape of a droplet whose surrounding space is filled with air. If we use a liquid with density ρ_L to substitute the air, then this liquid would reduce the gravity effect on the droplet shape. The modified Eq. (4) is as follows:

$$z = -\frac{|\rho-\rho_L|ga}{T'}\left[\left(y^2-\frac{y^3}{6a}-\frac{4}{3}ay\right)+k\left(y^2-2ay\right)\right], \tag{7}$$

where T' is the interfacial tension of the two liquids and k has the following form:

$$k = \frac{w}{2|\rho-\rho_L|ga}. \tag{8}$$

From Eq. (8), we find that by introducing ρ_L to the denominator helps to increase the k value. This allows us to increase droplet size without seeing the gravity effect. Such results are experimentally confirmed in Fig. 4. When $\rho=\rho_L$, $k\rightarrow\infty$, so that the gravity effect becomes negligible and the droplet size can be scaled up. Due to the negligible gravity effect, the droplet shape usually exhibits a spherical or parabolic profile when it is placed in vertical direction.

When the gravity does not take effect, the shape of the droplet should be axially symmetric. Because the surface of the droplet is very smooth, the liquid droplet usually exhibits a high lens performance with very small spherical aberrations. Fig. 7(a) shows the shape of a

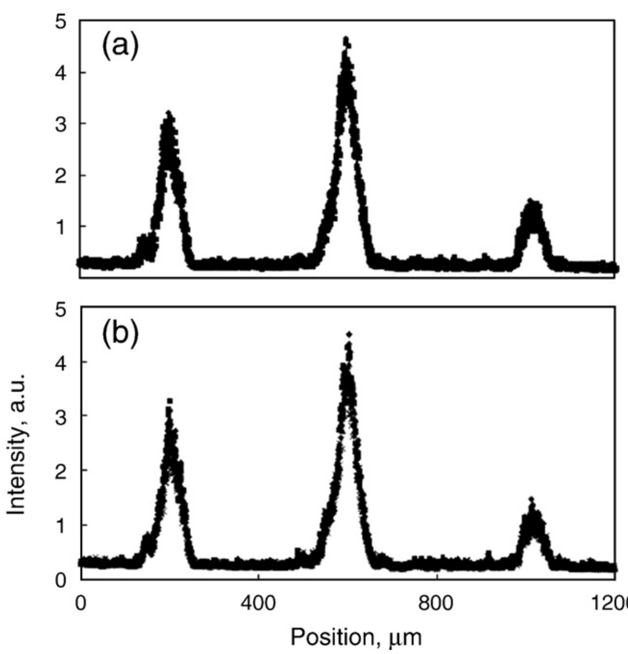

Fig. 6. The measured 2D spot intensity profiles across the spot center when the droplet was placed (a) in vertical direction, and (b) on a horizontal surface.

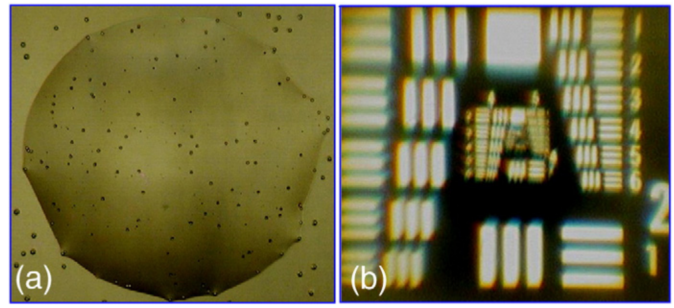

Fig. 7. (a) The shape of droplet covered with water and (b) the observed resolution bar through the droplet.

660-µm-aperture oil droplet surrounded with water in its outside space. There are some micro air bubbles attached on its surface and its aperture is not perfectly circular. However, we can still resolve 32 lp/mm, as shown in Fig. 7(b). Due to the low Abbe number (~22) of the oil material, color dispersion is the main reason to degrade the lens performance. When the droplet was placed upside down, the observed image (not shown) is the same as that in Fig. 7(b).

From our experiments, we find that when the gravity effect is negligible the droplet's shape is axially symmetric because of the surface tension, and the lens performance is not degraded by the gravity. If the outside space of the droplet is filled with another liquid rather than air, then the filled liquid helps the droplet to offset the gravity effect. Therefore, it is possible to scale up the droplet size with a negligible gravity effect. Under this circumstance, the densities of the two employed liquids need not to be matched.

5. Conclusion

We report the gravity effects on small liquid droplets. Simulation results agree well with our experimental results. For small droplets with a k value higher than 30, the effect of gravity on its shape is negligible. When the outside space of a droplet is filled with an immiscible liquid, then the k value is increased significantly. Therefore, gravity will not deform the shape of the droplet even though its size is increased. Based on our results, one can prepare microlens or microlens array using two immiscible liquids. The lens performances can be improved by choosing suitable materials without the concern of density matching.

Acknowledgments

H. Ren is supported by research funds of Chonbuk National University and UCF group is indebted to Areté Associates for partial financial support.

References

[1] A.W. Adamson, Physical Chemistry of Surfaces, Wiley-Interscience, 1960.
[2] M. Vallet, B. Berge, L. Vovelle, Polymer 37 (1996) 2465.
[3] C.A. Ward, M.R. Sasges, J. Chem. Phys. 109 (1998) 3651.
[4] B. Berge, J. Peseux, Eur. Phys. J. E 3 (2000) 159.
[5] T. Krupenkin, S. Yang, P. Mach, Appl. Phys. Lett. 82 (2003) 316.
[6] S. Kuiper, H.W. Hendriks, Appl. Phys. Lett. 85 (2004) 1128.
[7] S. Grilli, L. Miccio, V. Vespini, A. Finizio, S. De Nicola, P. Ferraro, Opt. Express 16 (2008) 8084.
[8] N.R. Smith, D.C. Abeysinghe, J.W. Haus, J. Heikenfeld, Opt. Express 14 (2006) 6557.
[9] C.C. Chen, C.A. Chang, J.A. Yeh, Opt. Express 14 (2006) 4101.
[10] H. Ren, H. Xianyu, S. Xu, S.T. Wu, Opt. Express 16 (2008) 14954.
[11] H. Ren, S.T. Wu, Opt. Express 16 (2008) 2646.
[12] S. Xu, J.Y. Lin, S.T. Wu, Opt. Express 17 (2009) 10499.
[13] Y.J. Lin, K.M. Chen, S.T. Wu, Opt. Express 17 (2009) 8651.
[14] S.A. Reza, N.A. Riza, Opt. Commun. 282 (2009) 1298.
[15] S. Vafaei, M.Z. Podowski, Nucl. Eng. Des. 235 (2005) 1293.
[16] N. Sugiura, S. Morita, Appl. Opt. 32 (1993) 4181.
[17] J. Pellicer, J.A. Manzanares, S. Mafé, Am. J. Phys. 63 (1995) 542.