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Design Optimization of Broadband Linear Polarization Converter Using Twisted Nematic Liquid Crystal

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An optimization method was developed for improving the designs of broadband linear polarization converters using a twisted nematic liquid crystal film and two uniaxial compensation films. As compared to the Poincaré Sphere approach, our optimization method takes the material dispersions into consideration and results in a broader bandwidth. Such a broadband half-wave film is particularly useful for enhancing the light efficiency of liquid crystal display devices.
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KEYWORDS: polarization converter, twisted liquid crystal film, optimization method, Jones matrix, Poincaré Sphere

Most liquid crystal display (LCD) devices require using linearly polarized light. However, the backlights developed for direct-view displays and arc lamps for projection displays are unpolarized. To convert an unpolarized backlight into linear polarization, a broadband reflective polarizer in conjunction with an achromatic quarter-wave film was proposed.[1,2] While for projection displays, a polarizing beam splitter array together with a broadband half-wave film was demonstrated.[3] The general requirements of such a polarization converter are broad bandwidth, thinness, light weight, and low cost.

The 90° twisted-nematic (TN) cell can serve as a broadband linear polarization converter (LPC).[4] The input linearly polarized light follows the molecular twist when the TN cell satisfies one of the following two criteria: the Mauguin limit ($\lambda \ll \Delta nd$)[5] or the Gooch-Tarry first minimum condition.[6] To satisfy the Mauguin limit requires a relatively thick LC layer. Especially, when a small birefringence polymeric film is considered, the required film is too thick to be conveniently fabricated. On the other hand, the Gooch-Tarry first minimum condition works only for a single wavelength. It is highly desirable to develop an achromatic linear polarization converter that can cover the entire visible wavelengths.

Recently, a linear polarization converter consisting of three liquid-crystal cells (two homogeneous cells and one twisted nematic (TN) cell) has been designed based on the Poincaré Sphere (PS) method.[7] Results are encouraging except that the bandwidth is inadequate.

In this letter, we describe a new method for optimizing the design of a linear polarization converter. In principle, the LPC can rotate the incoming linearly polarized light to a specifically designed angle, *e.g.*, 45°, 90°, etc. The most commonly used LPC is a half-wave plate. In this paper, we demonstrated a broadband half-wave plate using a TN cell and two uniaxial polymeric films as an example. By the same token, we could design an LPC with a specific rotation angle.

Figure 1 shows the device configuration of a broadband linear polarization converter. For the purpose of illustrating the calculation procedures, we used a twisted nematic cell sandwiched between two uniaxial compensation films. For practical applications, the LC cell should be replaced by a twisted LC polymeric film. These three films can be laminated together to form a compact and light weight half-wave plate. In our coordinate system, we define the rear rubbing direction of the TN cell as 0°. The incident linearly polarized light is at angle β and the device is designed to convert p-polarization to s-polarization or vice versa, which means that the polarization will be rotated by $\theta = \pi/2$, or the polarization of the output light is in the $\gamma = \beta + \pi/2$ direction. The TN cell has a twist angle ϕ. The optical axes of the two uniaxial polymer films are oriented at angles α_1 and α_2, respectively. The LC cell gap is assumed to be d and the thickness of the two films is d_1 and d_2, respectively.

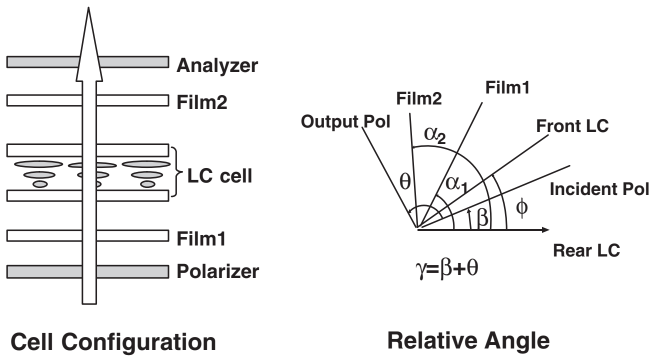

Fig. 1. Device configuration and orientation angles of the linear polarization converter.

In our simulations, we only calculated the normalized transmittance of the LPC; all the absorption and reflection losses from polarizer and substrates were ignored. The normalized transmittance for the linearly polarization component at an angle $\gamma = \beta + \pi/2$ can be obtained from the Jones matrix as:

$$T = \left| [\cos\gamma \quad \sin\gamma] M_{\text{film }2} M_{\text{LC}} M_{\text{film }1} \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix} \right|^2 \qquad (1)$$

where

$$M_{\mathrm{LC}} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos X - i(\Gamma/2)(\sin X/X) & \phi(\sin X/X) \\ -\phi(\sin X/X) & \cos X + i(\Gamma/2)(\sin X/X) \end{bmatrix}$$

$$M_{\mathrm{film\ 1}} = \begin{bmatrix} \cos\alpha_1 & -\sin\alpha_1 \\ \sin\alpha_1 & \cos\alpha_1 \end{bmatrix} \begin{bmatrix} \exp(-i\pi d_1 \Delta n_{\mathrm{film}}/\lambda) & 0 \\ 0 & \exp(i\pi d_1 \Delta n_{\mathrm{film}}/\lambda) \end{bmatrix} \begin{bmatrix} \cos\alpha_1 & \sin\alpha_1 \\ -\sin\alpha_1 & \cos\alpha_1 \end{bmatrix}$$

$$M_{\mathrm{film\ 2}} = \begin{bmatrix} \cos\alpha_2 & -\sin\alpha_2 \\ \sin\alpha_2 & \cos\alpha_2 \end{bmatrix} \begin{bmatrix} \exp(-i\pi d_2 \Delta n_{\mathrm{film}}/\lambda) & 0 \\ 0 & \exp(i\pi d_2 \Delta n_{\mathrm{film}}/\lambda) \end{bmatrix} \begin{bmatrix} \cos\alpha_2 & \sin\alpha_2 \\ -\sin\alpha_2 & \cos\alpha_2 \end{bmatrix}$$

Here, $X = \sqrt{\phi^2 + (\Gamma/2)^2}$ and $\Gamma = 2\pi d\Delta n/\lambda$. In our design, we have taken the LC and polymer material dispersions into consideration:[8)]

$$\Delta n = G\frac{\lambda^2 \cdot \lambda^{*2}}{\lambda^2 - \lambda^{*2}} \tag{2}$$

In eq. (2), G is a proportionality constant and λ^* is a mean resonant wavelength. To ensure a better phase matching between LC and polymeric films, similar molecular structures are recommended. A similar molecular structure leads to a similar λ^* so that the wavelength dispersion effect is compensated.[9)] For an LC and polymer containing a phenyl ring, their λ^* is in the vicinity of 210 nm. Let us assume that the LC employed has $\Delta n \sim 0.2$ at $\lambda = 600$ nm for liquid crystal and 0.033 for the polymer films. Thus, $G \sim 3.98 \times 10^{-6}\,\mathrm{nm}^{-2}$ and $0.663 \times 10^{-6}\,\mathrm{nm}^{-2}$ for the LC and polymer film, respectively. In the optimization method, the cost function is taken to be:

$$\mathrm{Cost} = -\int_{400\,\mathrm{nm}}^{700\,\mathrm{nm}} T(\lambda)\mathrm{d}\lambda \tag{3}$$

which is minimized in search of β, α_1, α_2, d, d_1 and d_2. The twist angle ϕ may be either fixed (such as specify $\phi = \pi/2$) or searched. The optimization method used in this work is known as the conjugate gradient method.[10)]

Figure 2 shows the simulation results for a 90° polarization rotator. This device is particularly important for display applications because it changes a p-polarization to s or vice versa. We obtain excellent results over the entire visible range. The optimal twist angle of the TN cell is found to be $\phi = 83.41°$ and cell gap $d = 5.825\,\mu$m. The incident wave is linearly polarized at $\beta = 9.18°$ and the angles [α_1, α_2] and film thicknesses [d_1, d_2] for the compensation film-1 and film-2 are [85.6°, −90.5°] and [2.48 μm, 4.96 μm], respectively. From Figs. 2(a) and 2(b), a small variation in cell gap (±0.1 μm) and twist angle (±1°) does not make any significant change in the performance of the rotator.

Figures 3(a) and 3(b) plot the thickness tolerance of films 1 and 2 for the 90° rotator, respectively. Again, the thickness tolerance of the two uniaxial films is quite acceptable.

To demonstrate the superiority of the 90° polarization rotator, we compared its optical transmission with the 90° TN LC cell between a pair of crossed polarizers and the device designed from the PS model.[7)] Results are shown in Figs. 4 and 5, respectively.

Figure 4 compares our results (gray lines) to those of 90° TN LC cells with $d\Delta n = 5\lambda$, 10λ, and 20λ. In all circumstances, the transmission oscillates with the wavelength. For the case of $d\Delta n = 5\lambda$, the transmittance oscillates slowly but produces a large amplitude variation

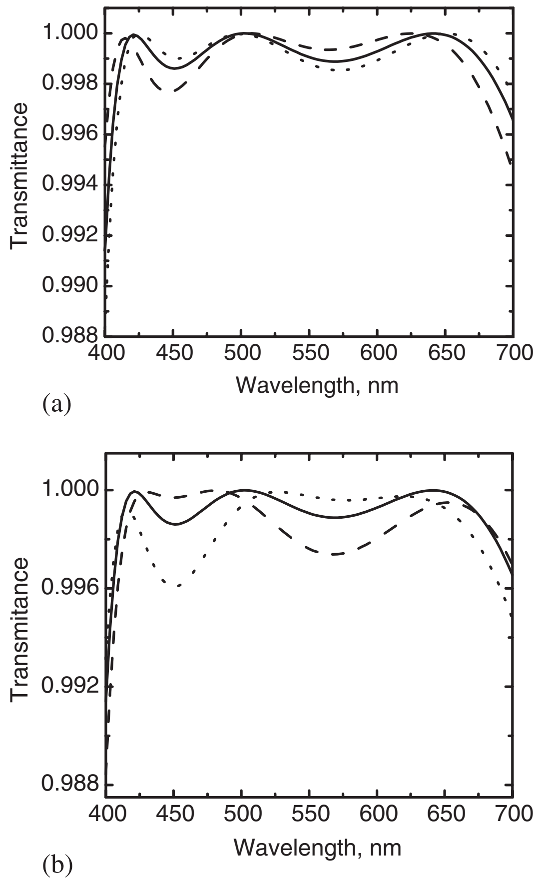

Fig. 2. (a) Cell gap tolerance of the 90° linear polarization rotator. Solid lines are for $d = 5.825\,\mu$m, dashed lines for $d = 5.9\,\mu$m and dots for $d = 5.7\,\mu$m. The rest parameters are the same: $\phi = 83.41°$, $\beta = 9.18°$, $\alpha_1 = 85.59°$, $\alpha_2 = -90.52°$, $d_1 = 2.48\,\mu$m and $d_2 = 4.96\,\mu$m. (b) Twist angle tolerance of the 90° linear polarization rotator. Solid lines are for $\phi = 83.41°$, dashed lines for $\phi = 84.41°$ and dots for $\phi = 82.41°$. The rest parameters are the same: $\beta = 9.18°$, $\alpha_1 = 85.59°$, $\alpha_2 = -90.52°$, $d = 5.83\,\mu$m, $d_1 = 2.48\,\mu$m and $d_2 = 4.96\,\mu$m.

as the wavelength changes. This indicates that the polarization rotation effect is very sensitive to wavelength, i.e., the Mauguin limit ($d\Delta n \gg \lambda$) is not satisfied. For the 90° TN cell with $d\Delta n = 10\lambda$, its result (dark solid line in Fig. 4) is comparable to, but still worse than ours at the longer wavelength region. As the $d\Delta n$ value increases to 20λ (dashed lines), the Mauguin limit is finally satisfied and the results are excellent. For a polymer film with $\Delta n \sim 0.03$, in order to satisfy $d\Delta n \sim 20\lambda$ in the visible region (say $\lambda \sim 550$ nm) the required film thickness is 367 μm. Such a thick twisted polymer film is difficult to manufacture. Therefore, our three film approach still has its technical

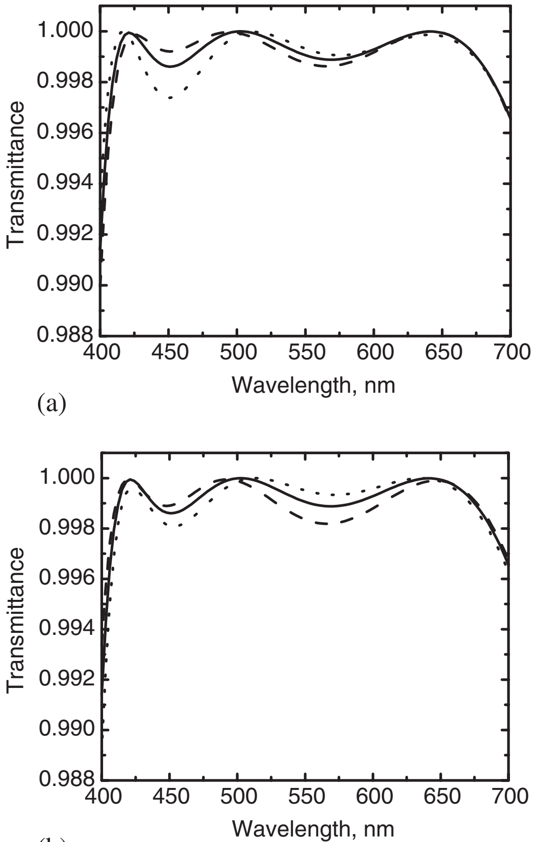

Fig. 3. (a) Thickness tolerance of compensation film-1. Solid lines are for $d_1 = 2.48\,\mu$m, dashed lines for $2.58\,\mu$m and dots for $2.38\,\mu$m. The rest parameters are the same: $\phi = 83.41^\circ$, $\beta = 9.18^\circ$, $\alpha_1 = 85.59^\circ$, $\alpha_2 = -90.52^\circ$, $d = 5.83\,\mu$m and $d_2 = 4.96\,\mu$m. (b) Thickness tolerance for film-2. Solid lines: $d_2 = 4.96\,\mu$m, dashed lines: $5.06\,\mu$m, and dots: $4.86\,\mu$m. The rest parameters are the same: $\phi = 83.41^\circ$, $\beta = 9.18^\circ$, $\alpha_1 = 85.59^\circ$, $\alpha_2 = -90.52^\circ$, $d = 5.83\,\mu$m, and $d_1 = 2.48\,\mu$m.

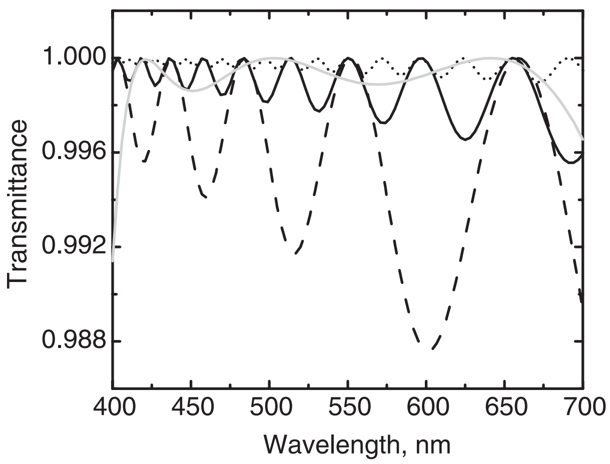

Fig. 4. Normalized transmission spectrum of a single 90° TN LC cell with different cell gap between crossed polarizers. Black dashed line: $d\Delta n = 5\lambda$, black solid line: $d\Delta n = 10\lambda$, black dot line: $d\Delta n = 20\lambda$, and grey solid line: the present design.

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Figure 5 compares our results with those obtained from the Poincaré Sphere model. The parameters used in the PS model are: LC birefringence $\Delta n = 0.2$, the thickness of the 90° TN cell $d = 1.9\,\mu$m and $\beta = 0^\circ$, and the two polymer

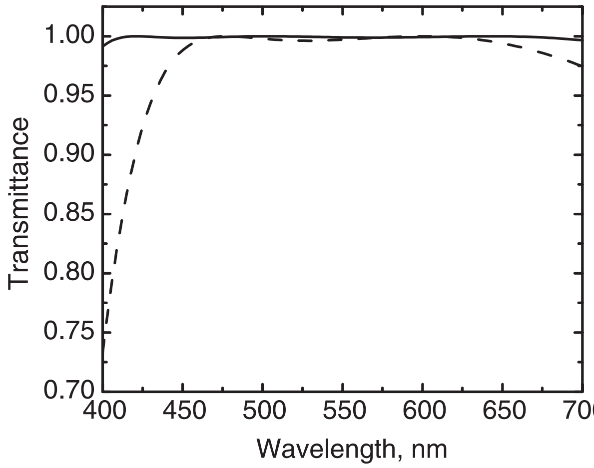

Fig. 5. Normalized transmission spectrum for a device obtained by the Jones method (solid line) and by the Poincaré Sphere method (dashed line).

films are oriented at -45° and 135°, respectively. According to the PS model, the phase retardations δ of the two films are identical and can be obtained from the following equation:

$$\tan\delta = \frac{2\lambda}{\Delta n p} \tag{4}$$

where λ is the wavelength, Δn is LC birefringence and $p = 2\pi d/\phi$ is the pitch, d and ϕ are LC cell gap and twist angle (here $\phi = \pi/2$), respectively. The thickness $d_{\rm film}$ of the two compensation films is calculated to be $1.93\,\mu$m with the use of eq. (4) and $\delta = 2\pi\Delta n_{\rm film} d_{\rm film}/\lambda$; here $\Delta n_{\rm film}$ is the birefringence of the two films. Figure 5 compares our results (solid lines) with those obtained from the PS model (dashed lines). Our design exhibits a wider bandwidth than that using PS method, especially in the short and long wavelength regimes.

In conclusion, we have demonstrated a novel design method for optimizing the device structure of a broadband linear polarization converter using a TN LC polymer film and two uniaxial compensation films. Our results show a wider bandwidth than that designed from the Poincaré Sphere method.

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