# Pretilt Angle Effects on Liquid Crystal Response Time

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***Abstract*—Pretilt angle effect on liquid crystal dynamics is analyzed theoretically. Analytical expressions are derived to describe liquid crystal response time under nonzero pretilt angle conditions. The theoretical analysis is confirmed experimentally using vertically aligned liquid crystal cells. This finding quantitatively correlates pretilt angles with liquid crystal response time. This study improves the understanding of the liquid crystal dynamic process which is helpful for optimizing liquid crystal response time.**

***Index Terms*—Liquid crystal (LC), pretilt angle, response time.**

## I. Introduction

LIQUID CRYSTAL (LC) response time plays a crucial role for display applications. A slow response time causes undesirable image blurring and should be avoided. LC response time is significantly influenced by the surface treatment of the substrates. A properly prepared substrate will orient the nematic LC directors in a preferred direction called pretilt angle [1]. Pretilt angle makes an important contribution to the dynamics of an LC cell [2]. However, detailed theoretical analysis has not been studied thoroughly.

In this paper, we derive analytical expressions for describing the LC dynamics including the pretilt angle effect. The analysis is valid for LC devices with pretilt angles, such as transflective displays with homogeneous alignment [3], [4] and LCoS displays with vertical alignment [5]–[7]. To confirm the theoretical analysis, we prepare several vertically aligned (VA) LC cells with various pretilt angles and measure their response times. In our experiments, we find that a large pretilt angle indeed greatly influences the LC response time. These experimental results are consistent with our theoretical analyses.

## II. Theory

Fig. 1 shows a VA nematic LC layer sandwiched between two parallel substrates where $z=-d/2$ and $+d/2$ stand for the bottom and top substrates, respectively. The $z$-axis is normal to the plane of the substrates, and the electric field E is along the $z$-axis. When the backflow and inertial effects are ignored, the Erickson–Leslie equation for describing the dynamics of LC directors is reduced to the following form [8], [9]:

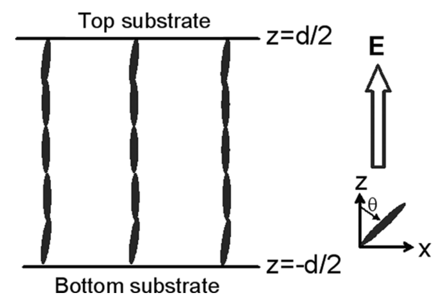


Fig. 1. Schematic drawing of a VA LC cell.

$$(K_{11}\sin^2\theta+K_{33}\cos^2\theta)\frac{\partial^2\theta}{\partial Z^2}+(K_{33}-K_{11})\times\sin\theta\cos\theta\left(\frac{\partial\theta}{\partial Z}\right)^2+\varepsilon_o\Delta\varepsilon E^2\sin\theta\cos\theta=-\gamma_1\frac{\partial\theta}{\partial t}. \quad (1)$$

In (1), $\gamma_1$ is the LC rotational viscosity, $K_{11}$ and $K_{33}$ represent the splay and bend elastic constants, respectively, $\varepsilon_{\rm o}\Delta\varepsilon E^2$ is the electric field energy density, $\Delta\varepsilon$ is the LC dielectric anisotropy, and $\theta$ is the tilt angle defined as the angle between the $z$-axis and the LC directors.

In general, (1) can only be solved numerically. However, when the tilt angle is small $(\sin\theta\sim\theta)$ (small angle approximation) [10] and $K_{33}\sim K_{11}$ (single elastic constant approximation), the Erickson–Leslie equation is simplified as:

$$K_{33}\frac{d^2\theta}{dz^2}+\varepsilon_o\Delta\varepsilon E^2\theta=-\gamma_1\frac{\partial\theta}{\partial t} \quad (2)$$

Equation (2) has following general solution:

$$\theta=[\theta_s\sin(\beta z)+\theta_m\cos(\beta z)]\cdot\exp(-t/\tau). \quad (3)$$

If the VA cells studied have the same top and bottom substrate treatments, then $\theta_{\rm s}$ is found to be 0. At a given voltage, $\theta_{\rm m}$ represents the maximum tilt angle in the center of the LC cell $(\theta|_{z=0}=\theta_m)$. If the top and bottom substrates have different alignment conditions, then $\theta_{\rm s}\neq 0$ and both terms in (3) have to be considered. Throughout this paper, for simplicity we assume the pretilt angles on both substrates are symmetric so that $\theta_{\rm s}=0$.

When the pretilt angle $\theta_{\rm p}$ is zero and the anchoring energy is strong, the following boundary conditions hold:

$$\theta_{Z=-\frac{d}{2},\frac{d}{2}}=\theta_p=0. \quad (4)$$

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Equations (3) and (4) lead to the following well-known analytical solutions for the decay $(\tau_d)$ and rise $(\tau_r)$ times:

$$\tau_d = \tau_o = \frac{\gamma_1 d^2}{K_{33}\pi^2} \tag{5}$$

$$\tau_r = \frac{\gamma_1}{\left|\varepsilon_o|\Delta\varepsilon|E^2 - \frac{\pi^2}{d^2}K_{33}\right|} = \frac{\tau_o}{\left|\left(\frac{V}{V_{th}}\right)^2 - 1\right|}. \tag{6}$$

In (5), $\tau_{\mathrm{o}}$ is called free relaxation time; i.e., during the decay process there is no bias voltage, and in (6) the threshold voltage is defined as

$$V_{th} = \pi\sqrt{\frac{K_{33}}{\varepsilon_o|\Delta\varepsilon|}}. \tag{7}$$

If the pretilt angle departs from zero, then we have:

$$\theta_{Z=-\frac{d}{2},\frac{d}{2}} = \theta_p \neq 0. \tag{8}$$

Equation (3) should satisfy the boundary conditions described by (8) at $Z = -(d/2)$ and $d/2$. From (3) and (8), we find the parameter $\beta$ has following form:

$$\beta = \frac{2}{d}\cos^{-1}\left(\frac{\theta_p}{\theta_m}\right). \tag{9}$$

Based on (2), we derive the new response time that takes pretilt angle effect into consideration:

$$\tau_d^* = \tau_o^* = \frac{\gamma_1}{\beta^2 K_{33}} \tag{10}$$

$$\tau_r^* = \frac{\gamma_1}{|\varepsilon_o|\Delta\varepsilon|E^2 - \beta^2 K_{33}|}. \tag{11}$$

In most cases, the maximum tilt angle is much larger than the pretilt angle, i.e., $\theta_{\mathrm{m}} \gg \theta_{\mathrm{p}}$. Under such a condition, the $\cos^{-1}()$ term in (9) can be approximated as:

$$\cos^{-1}\left(\frac{\theta_p}{\theta_m}\right) \sim \frac{\pi}{2} - \frac{\theta_p}{\theta_m} \tag{12}$$

and we derive the response time as follows:

$$\tau_d^* = \tau_o^* = \frac{\gamma_1}{\beta^2 K_{33}} = \frac{\gamma_1 d^2}{4K_{33}\left(\frac{\pi}{2} - \frac{\theta_p}{\theta_m}\right)^2} \tag{13}$$

$$\tau_r^* = \frac{\gamma_1}{\left|\varepsilon_o|\Delta\varepsilon|E^2 - \frac{4K_{33}}{d^2}\left(\frac{\pi}{2} - \frac{\theta_p}{\theta_m}\right)^2\right|}. \tag{14}$$

Strictly speaking, the LC threshold voltage $V_{\mathrm{th}}$ no longer exists if the pretilt angle is nonzero, although the threshold-like behavior in the voltage-dependent transmittance still appears. For simplicity, let us assume the threshold voltage still exists. Under such a condition, (14) can be simplified as:

$$\tau_r^* = \frac{\tau_o^*}{\left|\left(\frac{V}{\left(1-\frac{2\theta_p}{\pi\theta_m}\right)V_{th}}\right)^2 - 1\right|} \tag{15}$$

As expected, (13) and (15) are reduced to (5) and (6) when the pretilt angle is zero. Equations (13) and (15) suggest that the LC response time is also dependent on $\theta_{\mathrm{m}}$ which originates from the applied voltage. In [2], it is found that the pretilt angle effect becomes more pronounced when V gets close to $V_{\mathrm{th}}$. The derived expressions here confirm the bias voltage effect on LC response time, since $\theta_{\mathrm{m}}$ decreases when $V$ gets smaller.

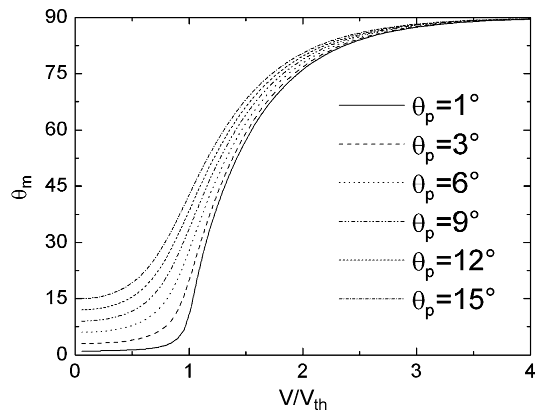


Fig. 2. Voltage dependent $\theta_{\mathrm{m}}$. The LC parameters used for simulations are: $\Delta\varepsilon = -4.2$, $\mathrm{V_{th}} = 2.19\ \mathrm{V_{rms}}$, $\mathrm{K_{11}} = 16.7$ pN, and $\mathrm{K_{33}} = 18.1$ pN.

Fig. 2 shows the simulation results of the voltage dependent $\theta_{\mathrm{m}}$. In the $V_{\mathrm{th}} < V < 4V_{\mathrm{th}}$ region, $\theta_{\mathrm{m}}$ increases significantly when the applied voltage increases and eventually approaches 90° at $V \sim 4V_{\mathrm{th}}$. Pretilt angles also influence $\theta_{\mathrm{m}}$, especially when $V$ is not too far above $V_{\mathrm{th}}$.

For an LCD device, the total response time is usually referred to the sum of rise and decay times. The rise time is strongly dependent on the applied voltage, and is usually much smaller than the decay time. With overdriving technique [11], the rise time can be further reduced. For this reason, the discussion in this paper will be focused on the LC decay process.

## III. Experiment

To confirm the theoretical analysis, we studied various VA LC cells which have different pretilt angles. For examples, two VA cells with the same rubbed polyimide were filled with two different negative LC materials. Both cells have strong anchoring energy ($> 4 \times 10^{-4}$ J/m$^2$) [12] so that the anchoring energy effect on LC response time can be neglected. Their pretilt angles and decay times were measured at room temperature ($T \sim 20$°C) and $\lambda = 633$ nm, respectively. It is known that pretilt angles are dependent on the LC materials even if the alignment conditions are the same [13], [14]. As a result, their pretilt angle effects on LC dynamics can still be different.

In experiments, the VA cells sandwiched between two crossed polarizers are biased at a voltage $V_{\mathrm{b}}$, which corresponds to the first transmittance maximum. Under such a condition, the total phase change is $\delta_0 = \pi$. When the bias voltage is released instantaneously at $\mathrm{t} = 0$, the time-dependent transmittance can be converted to the transient phase decay described by $\delta(\mathrm{t})$ [15]

$$\delta(t) \approx \delta_0 \exp(-2t/\tau_o) \tag{16}$$

From (16), $\tau_{\mathrm{o}}$ can be experimentally extracted from linear fitting of the time dependent $\ln(\delta_0/\delta(\mathrm{t}))$ curve.

In the first experiment, we filled a 6.97 $\mu$m VA LC cell with a negative LC material A, which is a modified MLC-6608 mixture from Merck. By numerical fitting of the experimental data, the pretilt angle was found to be 10.5°. Fig. 3 shows the

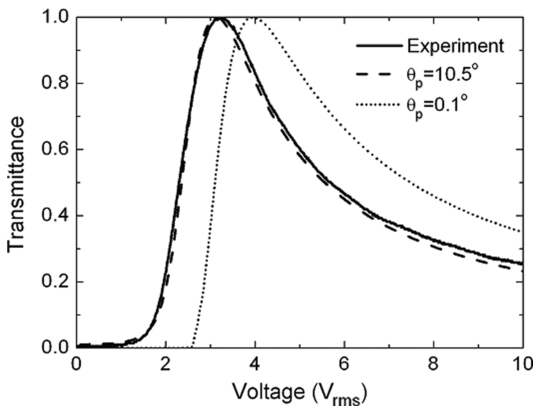


Fig. 3. Voltage dependent transmittance curves of a 6.97 $\mu\mathrm{m}$ VA cell at $\lambda = 633$ nm. The solid line is the experimental result, and dotted and dashed lines represent the simulation results for $\theta_{\mathrm{p}} = 0.1^{\circ}$ and 10.5°, respectively.

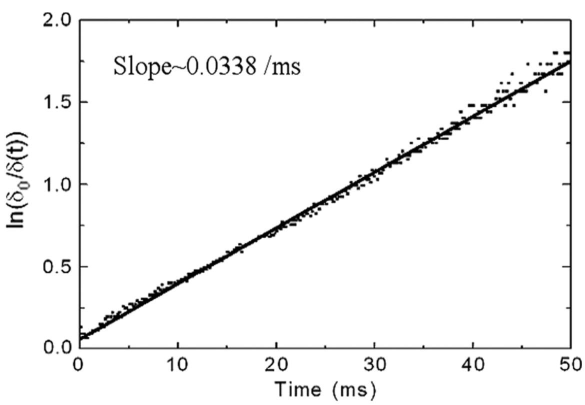


Fig. 4. Time-dependent $\ln[\delta_0/\delta(\mathrm{t})]$ of the 6.97 $\mu\mathrm{m}$ VA cell. Dots are experimental data and solid line is the fitting curve. The slope of the straight line is 0.0338/ms, and $\tau_{\mathrm{o}}$ is found to be ~59 ms.

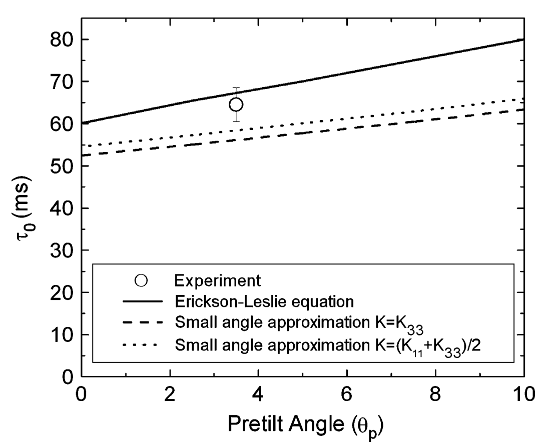


Fig. 5. Pretilt angle $\theta_{\mathrm{p}}$ dependent LC response time $\tau_{\mathrm{o}}$. Solid line represents the numerical solution of Erickson–Leslie equation [(1)]. The circle is the experimental result using LC mixture B. Dashed lines are the calculated results using (13) ($K = K_{33}$), which employs the small angle and one-elastic constant approximations. Dotted lines are also calculated from (13) except that $K_{33}$ is replaced by $K = (K_{11} + K_{33})/2$. Cell gap $\mathrm{d} = 7.10\ \mu\mathrm{m}$, and bias voltage $V_{\mathrm{b}} = 3.63\ V_{\mathrm{rms}}$.

voltage dependent normalized transmittance of the LC cell at $\lambda = 633$ nm and $T \sim 20^{\circ}$C. For comparison, numerically simulated curves under 10.5° and 0.1° pretilt angles are also plotted in the figure. A large pretilt angle smears the threshold behavior and lowers the effective threshold voltage. From Fig. 3, the maximum transmittance occurs at $V_{\mathrm{b}} = 3.25\ V_{\mathrm{rms}}$, which corresponds to $\delta_0 = \pi$. To measure decay time, we released the bias voltage and recorded the voltage dependent transmittance by a LabVIEW system. The transmittance data were converted to transient phase change as plotted in Fig. 4. A linear fitting of the experimental data leads to $\tau_{\mathrm{o}} \sim 59 \pm 2.5$ ms. If the pretilt angle effect is not considered, then $\tau_{\mathrm{o}}$ is calculated to be 42 ms based on (5). The measured experimental result is ~40.5% larger than the theoretical one. The discrepancy is rather significant. If we use the modified expression (13), the calculated result is $\tau_{\mathrm{o}} \sim 52$ ms, which is ~11.3% lower than the experimental data. In (13), $\theta_{\mathrm{m}} \sim 68^{\circ}$ is obtained from numerical simulation. By comparing the experimental and two theoretical results (including and excluding the pretilt angle effect), we find that the derived theoretical expression (13) describes the pretilt effect reasonably well. It indicates that the previously mentioned discrepancy mainly originates from the pretilt angle effect. Besides pretilt angle, backflow is another possible mechanism contributing to the discrepancy between the theoretical and experimental results. However, for a thin VA cell under low voltage operation the backflow effect should be relatively small.

In the second experiment, we tested a 7.10 $\mu$m VA cell, which has the same surface treatment as the first sample. The cell was filled with a commercial negative LC mixture B (MCL-6608). At 20°C, the LC parameters for mixture B are: $n_o = 1.4748$, $n_{\mathrm{e}} = 1.5578$ at $\lambda = 633$ nm, $\Delta\varepsilon = -4.2$, $\gamma_1 = 186$ mPas, $K_{11} = 16.7$ pN, and $K_{33} = 18.1$ pN. Similar to the first experiment, the pretilt angle was found to be 3.5° through fitting. Because of this smaller pretilt angle, the maximum transmittance occurs at 3.63 $V_{\mathrm{rms}}$ where $\delta_0 = \pi$. By the same method as the first experiment, $\tau_0$ was measured to be 64 ± 4 ms. Theoretical calculation based on (5) gives $\tau_{\mathrm{o}} \sim 52$ ms and the discrepancy is ~23%. When the 3.5° pretilt angle is taken into consideration, $\tau_{\mathrm{o}}$ is calculated to be 56 ms from (13), which is closer to the experimental result. Several other VA cells with various negative LC materials were also tested, and their pretilt angles are usually small ($< 2^{\circ}$). Under this circumstance, the effect of pretilt angle on the LC response time is insignificant. This result is also consistent with our theoretical analysis.

Numerical simulation based on finite-element method (FEM) is employed to solve (1), which avoids the "one-constant approximation" and "small-angle approximation" used in the theoretical analyses. In Fig. 5, the simulation results confirm that $\tau_{\mathrm{o}}$ increases as pretilt angle $\theta_{\mathrm{p}}$ gets larger, which is consistent with our theoretical analysis. Based on (13), two curves employing different elastic constant values, $K = K_{33}$ and $K = (K_{11} + K_{33})/2$, are also plotted. To deal the dynamics of VA cells, K $= K_{33}$ is usually used, but here the analytical results employing $K = (K_{11}+K_{33})/2$ is closer to the simulated curve. In (1), K $\sim$ $K_{33}$ is accurate only when $\theta$ is small. In a high voltage state, the LC directors tilt angle becomes relatively large so that the $K_{11}$ term is pronounced. Thus, it seems more reasonable to take $K_{11}$ into account and assume $K \sim (K_{11}+K_{33})/2$.

The discrepancy between the simulation and analytical results are mainly because of the single elastic constant ($K_{33} = K_{11}$) and small angle approximations. In the small pretilt angle region ($\theta_p \leq 2^\circ$), the difference is less than 11%. As $\theta_p$ increases, the discrepancy slightly increases. The result in our second experiment is represented by the circle in Fig. 5, and it agrees with the simulation and theoretical results reasonably well.

## IV. Conclusion

Pretilt angle is found to strongly influence the LC dynamics. Our theoretical analysis is confirmed by experimental and numerical simulation results. This finding improves the understanding of the LC dynamics. By optimizing the surface treatment and pretilt angle, LC response time can be improved.

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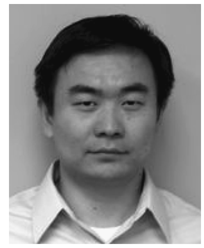

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Dr. Wu is a Fellow of SID and OSA.