Tunable-Focus Liquid Crystal Lens with Non-Planar Electrodes

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First published on: 19 August 2010

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We study a tunable-focus lens with convex and flat electrodes in a homogeneous liquid crystal (LC) cell. In a voltage-on state, a centro-symmetric gradient refractive index is generated within the LC layer, which causes the focusing behavior for the extraordinary wave. There have been experimental studies of the systems with spherical or parabolic shape of the top electrodes and there is a need in the theoretical modeling. We present a theoretical model of the lens with spherical top electrode and analyze its properties. We solve the Maxwell equations that are coupled to the Euler–Lagrange equations from the variational principle applied to the total free energy minimization. LC director spatial profile is found subject to the applied voltage. After that the phase retardation profile of the test light beam is calculated and thus a focal length dependence on the applied field. The simulation results agree well with experimental data. This LC lens has advantages in relatively large focal length range and low operating voltage.

Keywords Gradient index lens; non-planar electrode; tunable liquid crystal lens

1. Introduction

Liquid crystals are used in a variety of optical devices and in many applications, such as displays, deflectors, modulators and flaw detectors. Lenses with variable focal length are currently topics of broad interest. Tunable-focus lenses made on the base of liquid crystals [1–15] have many applications in optical communications, integrated optics, image and information processing, displays, optoelectronic devices and medicine. As focal length is driven by an electric field, there is no moving mechanical part and such lenses can be made smaller, lighter, and cheaper than the tunable lenses fabricated using conventional glass lenses.

The main principle of the lens operation is that the phase lag of the extraordinary wave is dependent on the position in the plane of the cell. As the liquid crystals are anisotropic media, the refractive index of the extraordinary wave depends on the liquid crystal director reorientation, which can be controlled with an external electric

field. If liquid crystal molecular reorientation changes with lateral position in the cell, then the phase lag will not be constant within the cell plane. If the director reorientation characteristics are radial symmetric, the cell can act as a lens.

Various designs for liquid crystal lenses have been developed. They differ in the methods used to create inhomogeneous director distribution. For example, the inhomogeneous electric fields inside the cell can be created by using electrodes with holes [1–5]. However, this cell configuration usually requires a relatively high voltage and its response time is fairly slow. This design is suitable for use as a microlens. Another subsequent lens design is based on Fresnel zones. Several ring electrodes on a cell wall form Fresnel zones [6–8]. The voltage across these electrodes changes slowly as a function of position. This scheme has been proposed as a design for optical spectacles. Its major advantages are twofold: low operating voltage and large optical focusing power (several diopters), but a disadvantage is that the resulting product is quite bulky. Inhomogeneous director reorientation and refractive index can be created by uniform electric field in the gradient polymer-stabilized liquid crystal (G-PSLC) with a spatially inhomogeneous polymer network [9–12]. Such tunable lens requires a low control voltage, but has a quite narrow dynamic range, i.e., its focal length change is limited.

In this paper, we consider a simple method for creating a LC lens, which is to use convex and flat tip electrodes in homogeneous LC cell. This LC lens exhibits a wide dynamic range, low control voltage, and good mechanical stability. These lenses use continuous electrodes to be easily manufactured, in comparison with patterned-electrode LC lenses. Many practical experiments [13–15] in this field have been performed but the theoretical analysis is lacking. The purpose of this paper is to perform the theoretical modeling of this class of lenses.

The arrangement of this paper is as follows. In §2 we describe the theoretical model. In §3 we discuss the LC directors reorientation in an inhomogeneous electric field. In §4 we examine the characteristics of the liquid crystal lens. Finally in §5 we draw some brief conclusions.

2. Model

Let us consider a nematic LC placed in a cell with thickness L and planar and uniform boundary conditions at each wall. We assume the surface anchoring is
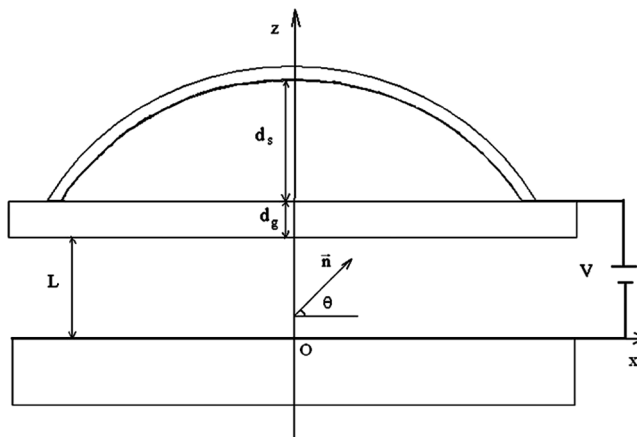

Figure 1. Liquid crystal lens cell structure. Point O is the coordinate origin.

strong at the cell walls. The geometry of the lens is shown in Figure 1. It consists of the LC cell and spherical glass shell, which contains an ITO-electrode on the inner surface. Another electrode is on the inner surface of the bottom substrate of the LC cell.

In Figure 1, the initial director orientation is along the x-axis. Oz is perpendicular to the cell walls, and an electric field is applied in this direction. Because the top electrode is spherical, the electric field in the cell is inhomogeneous $\vec{E} = (0, 0, E(\rho, z))$. We assume that the field reorients the LC directors in the xz-plane only. The refractive index profile varies as the field is applied. This gradient profile induces lens-like behavior; the focal length of such a lens is also field-dependent.

3. Director Reorientation

In order to investigate the director reorientation by an electric field we numerically minimize the free energy:

$$F = \frac{1}{2}[K_{11} \int (\nabla \cdot n)^2 dV + K_{22} \int (n \cdot \nabla \times n)^2 dV + K_{33} \int (n \times \nabla \times n)^2 dV] - \frac{1}{2} \int (D \cdot E)^2 dV, \tag{1}$$

For simplicity, we follow the one constant approximation, i.e., $K = K_{11} = K_{22} = K_{33}$ is the elastic constant. By radial symmetry, the director field is given by:

$$n = (\cos\theta(\rho, z), 0, \sin\theta(\rho, z)) \tag{2}$$

The Maxwell equations for the LC cell in electric field are:

$$\begin{cases} \nabla \cdot D = \dfrac{\partial \mathbf{D}_z}{\partial z} + \dfrac{1}{\rho}\dfrac{\partial(\rho \mathbf{D}_\rho)}{\partial \rho} = 0 \\ \nabla \times E = 0 \end{cases}. \tag{3}$$

The electric field in the z direction varies on a length scale $L \sim 10^{-5}$ m. By contrast, the scale of the inhomogeneity in the cell plane $\sim 10^{-3}$ m. The $\partial/\partial\rho$ derivatives in Eq. (3) can therefore be neglected in comparison with $\partial/\partial z$ derivatives. The rather complicated partial differential equations reduce to an easily solved set of ordinary differential equations with the radial coordinate ρ which can be regarded as a parameter.

By minimizing the free energy and simplifying the Maxwell Eq. (3), we obtain the system of equations to be solved:

$$\begin{cases} K\theta''_{zz} + \dfrac{E_z^2 \cdot \varepsilon_0 \varepsilon_\alpha}{2}\sin 2\theta = 0 \\ \frac{\partial}{\partial z}(D_z) = 0 \end{cases}. \tag{4}$$

In Euler–Lagrange equation of the system (4) we also neglect the $\partial/\partial\rho$ derivative as it is small comparing to $\partial/\partial z$ derivative; this assumption has been confirmed by numerical calculation in particular cases.

We also assume strong anchoring at the cell walls:

$$\theta|_{z=0} = \theta|_{z=L} = \theta_0. \tag{5}$$

The electric field in Eq. (4) can be rewritten in terms of the electric potential:

$$\vec{E} = -gradU, \ E_z = -\frac{\partial U}{\partial z}, \ D_z = \varepsilon_{zz}E_z = (\varepsilon_\perp + \varepsilon_a \sin^2 \theta)E_z \tag{6}$$

We assume that the electric potential is zero at the bottom electrode and gains a set of some values at the top electrode. At the other boundaries we consider the zero appropriate derivatives.
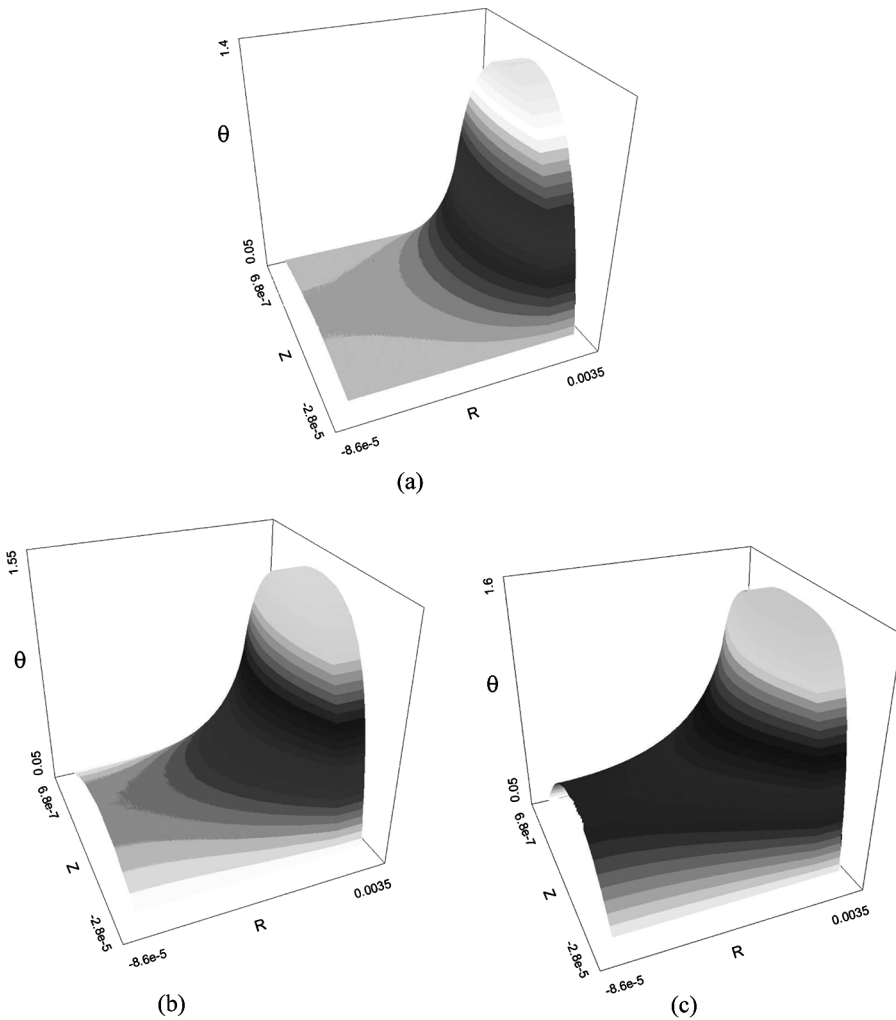

Figure 2. Inhomogeneous director reorientation angle at different voltage: (a) $V = 60\,V$, (b) $V = 100\,V$, and (c) $V = 160\,V$.

We solved Eq. (4, 6) with boundary conditions (5) numerically, using a finite element method [16]. For numerical calculation we need to set boundary conditions at all boundaries. We assume the zero appropriate derivatives in all the cases not mentioned above. Our model calculations use a cell thickness $L = 25\,\mu m$, pretilt angle at the cell walls $\theta_o = 6°$ and following LC parameters for BL-038: the principal components of the low frequency dielectric tensor $\varepsilon_{\parallel} = 19.7$, $\varepsilon_{\perp} = 5.3$, the principal refractive indices $\sqrt{\tilde{\varepsilon}_{\parallel}} = n_e = 1.799$, $\sqrt{\tilde{\varepsilon}_{\perp}} = n_o = 1.527$, and elastic constant $K \approx 10^{-11}\,N$. Geometric parameters of the lens are as following: $d_s = 0.72\,mm$ is the sag width, $d_g = 0.55\,mm$ is the glass width, $R = 3\,mm$ is the radius of the lens [14].

The LC director deviation profile for different values of applied voltage is shown in Figures 2 and 3, respectively. The electric field potential in the lens is shown in Figure 4 for $V = 100\,V_{rms}$.
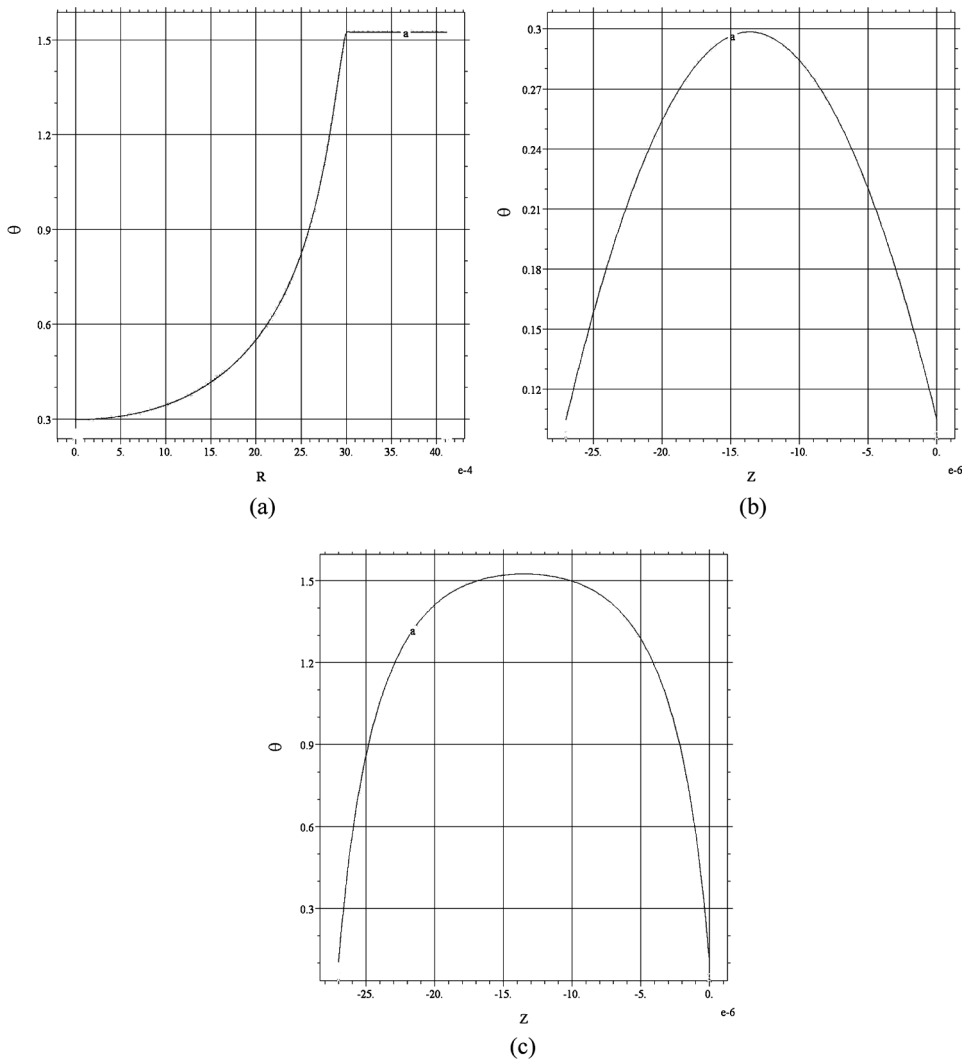

(a)

(b)

(c)

Figure 3. Director reorientation angle (a) in the middle of the LC layer along R-axis, (b) in the middle of the cell along z-axis, and (c) on the edge of the cell along z-axis.
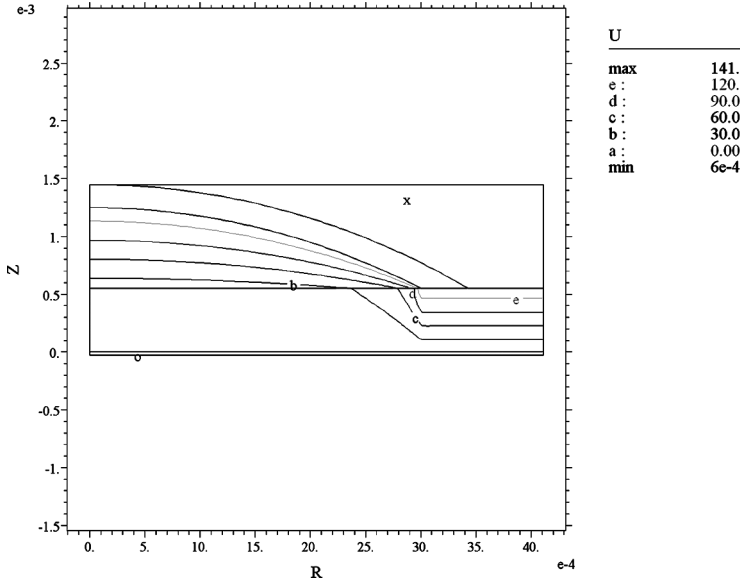
Figure 4. Electric field potential at applied voltage $V = 100\,V_{rms}$.

4. Focal Length

We now discuss the optical properties of the inhomogeneous liquid crystal cell. Lens geometry requires that the component of a test beam passing through the center of the lens has different phase lag from the component passing near the edge of the lens. In our system, a polarized test beam is similarly subject to a differential phase lag due to the generated gradient refractive index profile, thus we may say that an optical lens is created in the cell. Furthermore, since the phase lag is a function of external voltage, the lens properties are electrically controllable.

The phase lag of the ordinary wave is uniform across the cell, and no lens is therefore induced. So we confine our interest to the extraordinary wave. The phase lag profile of the extraordinary wave is given by the following formula [17]:

$$\varphi(\rho) = \frac{2\pi}{\lambda} \int_0^L n(\rho, z')dz', \tag{7}$$

where

$$n(\rho, z') = \frac{\sqrt{\tilde{\varepsilon}_\| \tilde{\varepsilon}_\perp}}{(\tilde{\varepsilon}_\| \sin^2 \theta + \tilde{\varepsilon}_\perp \cos^2 \theta)^{1/2}} \tag{8}$$

is the refractive index for the extraordinary beam propagating in the LC cell, $\theta(\rho, z')$ is the director deviation calculated in §3, $\hat{\tilde{\varepsilon}}$ is the dielectric tensor at optical frequency.

To determine the effective focal length of the lens we fit (7) to a parabola [18]:

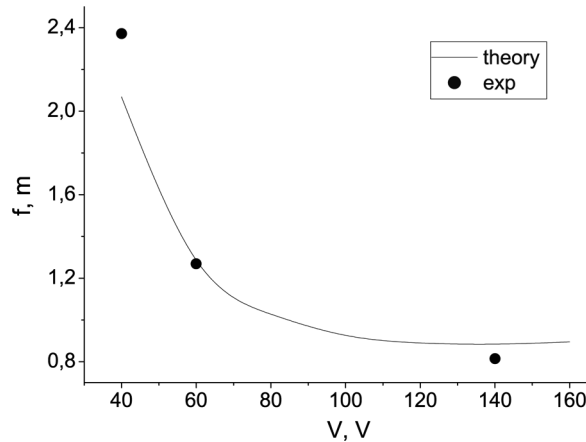$$\varphi(\rho) = a - \frac{k\rho^2}{2f}, \tag{9}$$

Figure 5. Focal length dependence on the applied voltage. Solid line is the result of theoretical calculations and dots are experimental results [14].

where $a = \varphi(0)$ is the phase shift at the center of the lens and k is the absolute value of the wave vector of the test beam. Then the focal length is given by the best estimator to f.

In Figure 5 we plot the dependence of the focal length on the applied voltage. The agreement between theoretical model and experimental results is fairly good.

5. Conclusion

We have presented a theoretical study of a tunable-focus lens using a glass shell and a homogeneous LC layer. Minimizing the total free energy functional in the presence of an external voltage, we calculated numerically the LC director profile in the cell subject to externally applied electric field. Due to the electrode curvature the electric field in the LC is inhomogeneous and radial symmetric. We have used a Fresnel approximation to determine phase shifts for the extraordinary beam, and hence determine the lens' focal length. The focal length of this LC lens decreases with increasing voltage. The calculated results are in good agreement with experimental data. Our results enable improved quantitative understanding of the lens and thus enable an improved design process for the device parameters. In future work we shall consider in more detail specific lens parameters.

Acknowledgments

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