Particle swarm optimization-based approach for optical finite impulse response filter design

Ying Zhou, Guangjie Zeng, and Feihong Yu

This study presents what is to our knowledge a new and efficient method for the design of an optical finite impulse response (FIR) filter by employing a particle swarm optimization technique. With the method proposed, the design of an optical FIR filter, which is able to provide an arbitrary spectrum output based on crystal birefringence, could be implemented with good performance and high efficiency. The design procedure is discussed. A typical example of a green/magenta filter used in a liquid crystal on silicon projection display is included to demonstrate the feasibility and efficiency of this method in this design process as compared with simulated annealing. © 2003 Optical Society of America

OCIS codes: 120.2440, 120.4570, 120.4640, 260.1440, 350.2460.

1. Introduction

During the past few decades, a birefringent filter proved to be an effective tool in a number of applications, such as astronomical observation, tunable lasers, optical communications, and projection display. There are four previously known types of birefringent filters[1]: Lyot, partial polarizing, Solc,[2] and multiple liquid-crystal tunable filters.[3] Among them, the regular structures of the first three types result in a relatively simple output. Usually, only the narrowband spectrum is available. While the multiple liquid-crystal tunable filter suffers the main drawback of being difficult to control, in contrast, the optical finite impulse response (FIR) filter exhibits its own attractiveness in providing an arbitrary spectrum with double channels to meet the increasing needs and serving not only as a monochromator but also as a color manipulator over an extended field.[4] Derived from the Solc filter, the optical FIR filter consists of a stack of identical retardation plates of birefringent material sandwiched between two linear polarizers, one at each end. But the relative angles of each plate are arranged in a specific configuration determined by the optimization algorithm. This structure determines the output that if N plates are concerned, there will be $N + 1$ impulses with an identical time interval, coincident with the FIR filter. So it is named the optical FIR filter.[5]

Briefly, the basic idea for the optical FIR filter design is to: first develop the desired output spectrum into the finite terms of a Fourier series and then determine the whole structure (the relative angles of both retarders and analyzer) according to the optical network backward transfer method proposed by S. E. Harris.[6] How to approximate the Fourier series with the desired output spectrum in minimal terms is the key problem. The evaluation function parameterized with retarder angles is quite complex with multiple peaks. For such a problem as multiparameter optimization, global optimization algorithms such as a genetic algorithm have been attempted feasibly but the genetic operators, such as selection, crossover, and mutation are relatively complex. By comparison, simulated annealing (SA) is much simpler but it needs too many iterations, and the result strongly depends on the cooling schedule preestablished.[8] Here, we employ the particle swarm optimization (PSO) technique as proposed by Kennedy and Eberhart in 1995[9] as an alternative method to implement optical FIR filter design. We have achieved good performance with high efficiency.

The PSO technique, a kind of evolution computation, with roots in the preying of large number of birds or fish. The underlying rules of cooperation and competition within social swarms give it good capability to make global optimization with the help of memory rather than to simply random search in a certain area. So it has a better chance to fly into a better solution quickly than some previous optimiz-

The authors are with the Optical Engineering Department, State Key Laboratory of Modern Optical Instrumentation, Zhejiang University, Hangzhou, China, 310027.

Received 17 June 2002; revised manuscript received 14 October 2002.

0003-6935/03/081503-05$15.00/0

© 2003 Optical Society of America

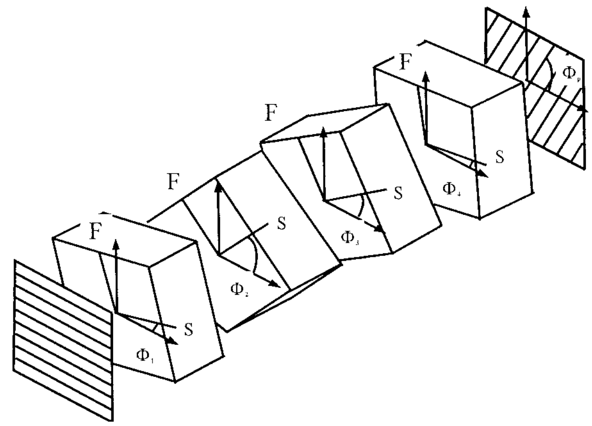

Fig. 1. Basic structure of optical FIR filter.

ers have in addition to its better performance. This paper concentrates on the application of PSO to design an optical FIR filter. The example of broadband green/magenta (G/M) filter used in a liquid crystal on silicon (LCOS) projection display for color separation/combination follows to demonstrate its effectiveness. The efficiency in optical filter design compared with the SA algorithm is also given.

2. Optical Finite Impulse Response Filter

The optical FIR filter is composed of a stack of identical birefringent plates arranged in specific angles. The whole stack is then placed between a linear polarizer and an analyzer. All the birefringent crystals are cut with their crystal optical axes parallel to the surfaces, so that they act as the retardation plates.[6,7]

As shown in Fig. 1, the polarized light after the polarizer comes through the retarder stack, in which the retarders are of the same thickness and crystal material. Because of birefringence, each retarder separates the input light into two components along its fast and slow axis respectively, and each component acts as the input of the next one. Therefore, if there are N plates, it will produce 2^N impulses. In fact, there are only $N + 1$ impulses due to the identical phase delay brought from identical plates. The time interval between them is determined by the crystal thickness and the difference of the material refractive indices. Eq. (1) shows this output.

$$\begin{aligned} C(t) &= C_0\delta(t) + C_1\delta(t-a) + C_2\delta(t-2a) \\ &\quad + \cdots + C_n\delta(t-na) \\ &= \sum_{k=0}^{n} C_k\delta(t-ka), \end{aligned} \tag{1}$$

where, a denotes the time interval of the impulse series, given by

$$a = t_s - t_f = \frac{L\Delta\eta}{c}. \tag{2}$$

Here, t_s, t_f are the time that two components along the fast and slow axes needed to pass through a single plate. c is the velocity of light in vacuum and $\Delta\eta$ represents the difference of their refractive indices.

$$\Delta\eta = n_e - n_o. \tag{3}$$

So the phase delay caused by a single plate can be expressed as (if we suppose that λ is the wavelength in vacuum)

$$\Gamma = \frac{2\pi\Delta\eta L}{\lambda}. \tag{4}$$

The frequency response of the filter is the Fourier transform of Eq. (1), expressed as

$$\begin{aligned} C(\omega) &= C_0 + C_1 \exp(-ia\omega) + C_2 \exp(-i2a\omega) \\ &\quad + \cdots + C_n \exp(-ina\omega) \\ &= \sum_{k=0}^{n} C_K \exp(-ika\omega), \end{aligned} \tag{5}$$

where $\omega = 2\pi f = 2\pi c/\lambda$ denotes the angular frequency of the light. Hence the spectrum response of the filter is the equivalent transform of Eq. (5).

$$\begin{aligned} C(\lambda) &= C_0 + C_1 \exp\left[-i2\pi\Delta\eta L\left(\frac{1}{\lambda}\right)\right] \\ &\quad + C_2 \exp\left[-i4\pi\Delta\eta L\left(\frac{1}{\lambda}\right)\right] \\ &\quad + \cdots + C_n \exp\left[-in2\pi\Delta\eta L\left(\frac{1}{\lambda}\right)\right] \\ &= \sum_{k=0}^{n} C_k \exp\left[-ik2\pi\Delta\eta L\left(\frac{1}{\lambda}\right)\right] \\ &= \sum_{k=0}^{n} C_k \exp(-ik\Gamma). \end{aligned} \tag{6}$$

From the discussion above, it can be seen that the indirect design method is applied here. We should first optimize the coefficients C_k to approximate the desired spectrum, and then convert them into the series of actual angles according to optical network backward transfer technique. Because all of the detectors receive only light intensity, what they actually receive is $C(\lambda)^2$. The complementary color, which is the output along the perpendicular direction of the analyzer, can be expressed as $D(\lambda)^2$.

$$D(\lambda) = \sum_{k=0}^{n} D_k \exp\left[-ik2\pi\Delta\eta L\left(\frac{1}{\lambda}\right)\right]. \tag{7}$$

In optical network backward transfer, the angle of each plate is deduced backward according to the principle of energy conservation and the relationship between the output and input of each plate. The details for computation can be found in Ref. 6.

3. Application of Particle Swarm Optimization in an Optical FIR Filter Design

A. Particle Swarm Optimization

Particle swarm optimization simulates the social behavior of a flock of birds or fish.[9–12] Just as the other evolutionary computation, there are a population of individuals (particles). Each individual, in a concrete problem, represents a potential solution and is taken as a point in a D-dimension problem space. The position of ith particle is noted as $pX_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$ with fitness pfitness and velocity $V_{id} = (v_{i1}, v_{i2}, \ldots, v_{iD})$. Its previous best position (in the sense that the position gives the best fitness) is noted as $P_i = (p_{i1}, p_{i2}, \ldots, p_{iD})$ and the corresponding fitness is pbest. The best fitness of this population within this iteration is recorded as pfitness_best. The best previous position in the whole population is $gX = (p_{g1}, p_{g2}, \ldots, p_{gD})$ with fitness gbest. n denotes the number of iterations. Within each generation, the particle renews its own velocity and position according to two rules as Eq. (8) and Eq. (9) show:

$$v_{id} = w(n)v_{id} + C_1\text{rand}(\,)(p_{id} - x_{id}) + C_2\text{Rand}(\,)(p_{gd} - x_{id}), \quad (8)$$

$$x_{id} = x_{id} + v_{id}, \qquad d = 1, 2, \ldots, D. \quad (9)$$

Usually $C_1 = C_2 = 2$ is chosen to take the same weight. rand( ) and Rand( ) are random factors uniformly distributed between 0 and 1. In each generation, after the particle renews its velocity and position, the characteristic parameters of the population, such as P_i, pbest, gX, and gbest are refreshed. To avoid oversized velocity, the velocity should be confined within a certain value and the location should be limited within the area of the solutions. The calculation of fitness is discussed in Subsection 3.B.

The inertia weight function

$$w(n) = \frac{0.5n}{1 - N} + \frac{0.4 - 0.9N}{1 - N} \quad (10)$$

will decrease linearly from 0.9 to 0.4 through the run to adjust the global and local searching capability. N is the maximal times of iteration. The bigger w is, the stronger the global searching ability it has. With the initial position and velocity of each particle produced randomly by computer, the basic process of particle swarm search is diagramed on Fig. 2.

It is noted that Eq. (8) is composed of three parts. The first part provides the particle with the tendency to exploit new areas. The second part gives the particle self-cognition, while the third part makes the particle show social-cognition. All of them contribute and share with the memory of the whole population. In other words, the particle updates itself constantly according to the information both from itself and from the entire population. Therefore it is more likely to fly into the optimal solution area rapidly.

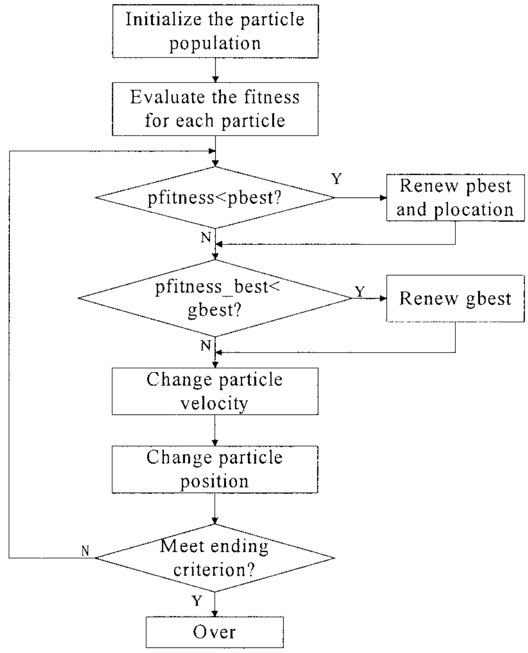

Fig. 2. Basic process of particle swarm optimization.

B. Optical FIR Filter Design with Particle Swarm Optimization

To apply the optimization to the optical FIR filter design problem, a reasonable evaluation function is prerequisite. Given that the desired spectrum distribution is $C_{\text{desired}}(\omega)$, the actual spectrum distribution is $C_{\text{actual}}(\omega) = C(\omega)C(\omega)^*$. The evaluation function could be designed in the sense of weighted least square with the variables of C_K, expressed in Eq. (11).

$$F_{\text{obj}}(C_K) = \sum_{i=0}^{i=M} P(\tilde{\omega}_i)W(\tilde{\omega}_i)[C_{\text{actual}}(\tilde{\omega}_i, C_K) - C_{\text{desired}}(\tilde{\omega}_i)]^2, \quad (11)$$

where

$$\tilde{\omega}_i = \frac{i}{M}, \quad i = 0, 1, 2 \ldots M. \quad (12)$$

It represents the normalized sample frequency in one basic period of the filter. Here $P(\tilde{\omega}_i)$ is the penalty (weight) of a single point that is used to restrict ripples on this point.

$$W(\tilde{\omega}_i) = \begin{cases} w_1 \;, & \text{within pass band} \\ w_2 \;, & \text{within stop band} \\ w_3 \;, & \text{within trans\_band} \end{cases} \quad (13)$$

represents the weight of each band. trans_band is a user-defined variable representing the transition bandwidth. It works as a control parameter to enable the tradeoff of ripples and transition bandwidth. If trans_band is too small, the object function will

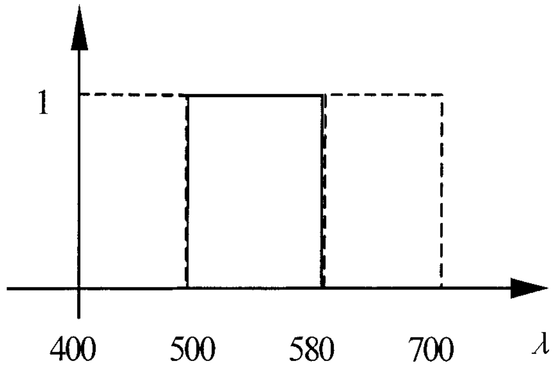

Fig. 3. Desired spectrum of G/M broadband filter.

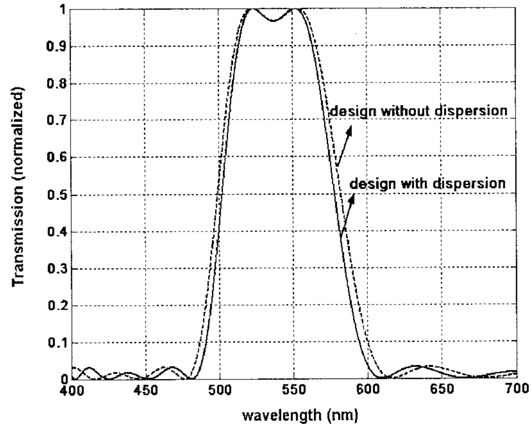

Fig. 5. Comparison of design with dispersion and without dispersion.

inevitably optimize the curve in the minimal transition bandwidth so as to cause large ripples. Therefore, the reasonable definition of trans_band will contribute to suppressing ripples. Usually, we choose w_1, $w_2 > w_3$ to loosen the requirement of the curve in the transmission band.

Unlike other algorithms, PSO has few parameters to adjust and the result has little relationship with the size of the population, which makes the filter design easy to operate.

4. Example Design and Comparison

Here, the example of a G/M broadband filter design used in the LCOS projector optical engine for color separation/combination is presented to demonstrate the effectiveness of this method. The desired spectrum is shown in Fig. 3. The solid line denotes the green color spectrum and the dashed line denotes its complementary color, magenta. Either color could be easily obtained through 90° rotation of the analyzer or the polarization beam splitter.

With randomly produced initial particles' velocities and positions, we get the result of a six-order filter by the PSO technique, shown as Fig. 4. This result is obtained under the condition that the maximal times of iteration $N = 4000$, the population size popsize $=$ 20, and the number of sample frequency point $M =$ 256. In Fig. 4, the solid curve shows the design with PSO and the dashed curve shows the design with SA.

It seems that the two designs have slight differences between them.

In practical use, the spectrum of the filter will be affected by the dispersion of material. For material of quartz, whose dispersion coefficients are relatively smaller, the spectrum of the G/M filter will appear as shown on Fig. 5. The solid curve shows the design in consideration of material dispersion and the dashed curve shows the former design without dispersion, the same as the curve shown by the solid curve in Fig. 4.

Although both the two algorithms are able to find satisfying results, their efficiencies to find an optimal solution are extraordinarily different. Here, we show the convergent process of these two algorithms from which we can learn the causes. First, Fig. 6 depicts the decreasing process of the best fitness of 20 particles (gbest) in only 1000 iterations, though the final iteration is 4000. Fig. 7 depicts how fitness in SA changes during the run, in which each iteration refers to one completion of the Markov chain with a length of 3000 iterations. In the process of SA, the optimal solution is obtained after 68 iterations. So the times of evaluation completed by the PSO versus the SA are $4000 \times 20{:}3000 \times 68$. Second, for the same evaluation function, the average time consumed by the PSO in a whole run is approximately

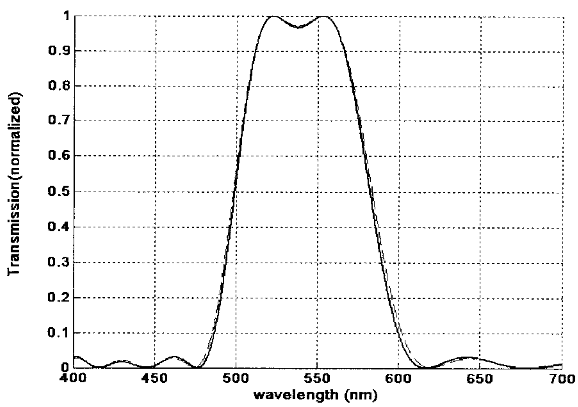

Fig. 4. Comparison of G/M filter design in PSO and SA, six order.

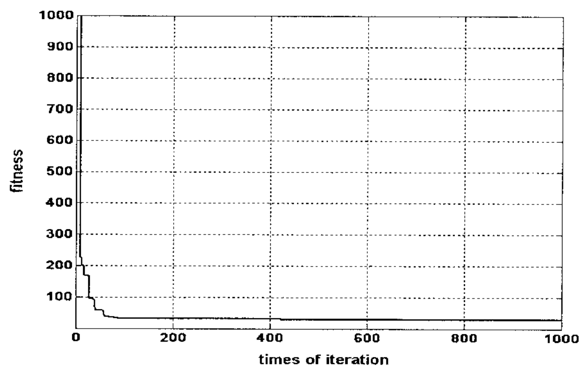

Fig. 6. Decrease process of PSO.

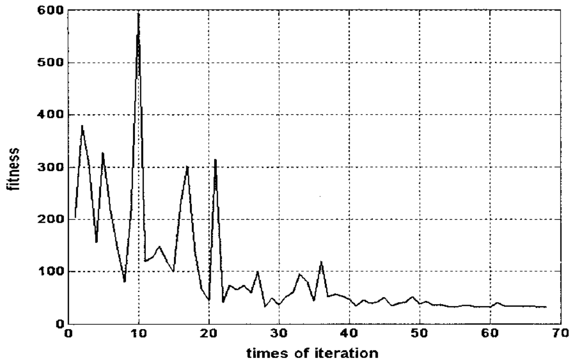

Fig. 7. Decrease process of SA.

2.5 min but in contrast the time consumed by SA is over 8 min. This is tested in a C program, under PIII650 CPU, 256M Ram. Third, the decrease in the PSO is monotonic, while the decrease in the SA is oscillatory, the characteristic features of the two algorithms. Tracking the minimal fitness of the two algorithms, we can see that PSO converges much faster than SA in this problem. Fourth, there are fewer parameters to adjust in the PSO, while in the SA the parameters in the cooling rate have large impact on the results. Thus, the PSO is easier to operate. Concerning time consumption and convergence speed, the PSO is more efficient than the SA in this particular problem of optical FIR filter design.

5. Conclusions

A simple and effective method for optical FIR filter design based on particle swarm optimization is presented. Its attractiveness comes from its high efficiency with few parameters to adjust for good performance. For optical FIR filter design, particle swarm optimization is effective to attain arbitrary spectrum output.[13]

This project is supported by the National Natural Science Foundation of China and the Joint Foundation of Ningbo Science & Technology Commission and Zhejiang University. The authors thank H. S. Kwok at Hong Kong University of Science and Technology for his helpful suggestion.

References

1. A. M. Titile and W. J. Rosenberg, "Tunable birefringent filters," Opt. Eng. **20,** 815–823 (1981).
2. J. W. Evans, "Solc birefringent filter," J. Opt. Soc. Am. **48,** 142–146 (1958).
3. Y. Kong, G. H. Li, M. J. Yun, X. Y. Fan, Y. T. Pang, and N. Wang, "A research on multiple tunable liquid crystal filter," Journal of Optoelectronics · Laser **12,** 474–476 (2001).
4. Y. Zhou, F. H. Yu, and H. S. Kwok, "Optical FIR filter design based on crystal birefringence," Opto-Electronic Engineering, to be published (2002).
5. Z. Y. He, Digital Signal Processing: Theory and Application (People's Posts & Telecommunications Press, Beijing, 1983) pp. 189–333.
6. S. E. Harris, E. O. Ammann, and I. C. Chang, "Optical network synthesis using birefringent crystals. I. Synthesis of lossless network of equal-length crystals," J. Opt. Soc. Am. **54,** 1267–1279 (1964).
7. G. D. Sharp and J. R. Birge, "Retard Stack Technology for Color Manipulation," SID Symposium **30,** 1072–1075 (1999).
8. Y. Zhou, G. Z. Wu, and F. H. Yu, "Applications of memory simulated annealing for optical FIR filter design," Acta Optica Sinica, to be published, (2002).
9. R. C. Eberhart and J. Kennedy, "A new optimizer using particle swarm theory," in *Proceedings of the Sixth International Symposium on Micro Machine and Human Science.* (Piscataway, NJ: IEEE Service Center, 1995), pp. 39–43.
10. J. Kennedy and R. C. Eberhart, "Particle swarm optimization," in *Proceedings of the IEEE International Conference on Neural Networks, IV.* (Piscataway, NJ: IEEE Service Center, 1995), pp. 1942–1948.
11. Y. H. Shi and R. C. Eberhart, "Empirical study of particle swarm optimization," in *Proceedings of the 1999 Congress on Evolutionary Computation.* (Piscataway, NJ: IEEE Service Center, 1999), pp. 1945–1950.
12. Y. H. Shi and R. C. Eberhart, "A modified particle swarm optimizer," in *Proceedings of the IEEE International Conference on Evolutionary Computation.* (Piscataway, NJ: IEEE Service Center, 1998b), pp. 69–73.
13. Y. B. He, J. Y. Xiong, W. Zheng, G. J. Zeng, and F. H. Yu, "Technology and applications of LCOS projector," Optical Instruments **24,** 38–46 (2002).